

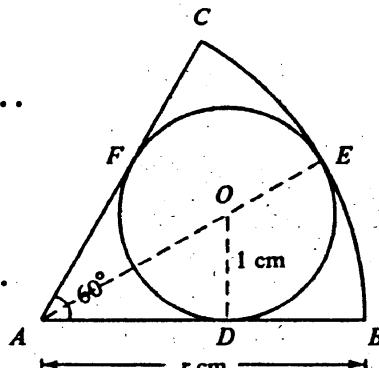
SOLUTION	MARKS	REMARKS
<p>1. (a) $x^2 - 2x + 1 = (x - 1)^2$</p> <p>(b) $x^2 - 2x + 1 - 4y^2 = (x - 1)^2 - 4y^2$ $= (x - 1 - 2y)(x - 1 + 2y) \dots$ $= (x - 2y - 1)(x + 2y - 1)$</p>	2A 1M 1M+1A <u>5</u>	or $(x-1)(x-1)$ for $(\quad)^2 - 4y^2$ 1M for diff. of 2 sq's. No marks for $x^2 - 4y^2 = (x-2y)(x+2y)$
<p>2. Let $f(x) = 2x^3 + ax^2 + bx - 2$</p> <p>Putting $x = 2$, $f(2) = 4a + 2b + 14$</p> <p>As $x - 2$ divides $f(x)$, $4a + 2b + 14 = 0$.</p> <p>Similarly</p> <p>$f(-1) = a - b - 4$ $= 0$</p> <p>Solving the equations, $6a + 6 = 0$ $a = -1, b = -5$</p>	1A 1M 1A 1A+1A <u>5</u>	for $f(2) = 0$ or $f(-1) = 0$
<p>(Syll A)</p> <p>3. (a) $\sqrt{\frac{3^{5k+2}}{27^k}} = \sqrt{\frac{3^{5k+2}}{(3^3)^k}}$ $= 3^{k+1} \dots$</p> <p>(b) $\frac{\log a^3 b^2 - \log a b^2}{\log \sqrt{a}} = \frac{\log \frac{a^3 b^2}{ab^2}}{\log \sqrt{a}} \dots$ $= \frac{\log a^2}{\log \sqrt{a}}$ $= \frac{2 \log a}{\log a} \dots$ $= 4$</p>	1A 1A 1A 1A 1A <u>5</u>	or $= \frac{\log a^3 + \log b^2 - \log a - \log b^2}{\log \sqrt{a}}$ $= \frac{3 \log a - \log a}{\log a}$ 1A
<p>(Syll B)</p> <p>3. $3^{2x} + 3^x - 2 = 0$ $(3^x)^2 + 3^x - 2 = 0 \dots$ $(3^x - 1)(3^x + 2) = 0$ $3^x = 1 \text{ or } 3^x = -2$ (Rejecting $3^x = -2$) $x = 0$</p>	1M 1A 1A 1A 1A <u>5</u>	$(3^x)^2$) Accept $3^x = 1$)

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87 MATHS (SYLL A/B)

	SOLUTION	MARKS	REMARKS
4.	$\sin^2 \theta = \frac{3}{2} \cos \theta$ $1 - \cos^2 \theta = \frac{3}{2} \cos \theta$ $2\cos^2 \theta + 3\cos \theta - 2 = 0$ $(2\cos \theta - 1)(\cos \theta + 2) = 0$ $2\cos \theta = 1 \text{ or } \cos \theta = -2$ Rejecting $\cos \theta = -2$, we have $\cos \theta = \frac{1}{2}$ $\theta = 60^\circ \text{ or } 300^\circ \text{ (or } \frac{\pi}{3}, \frac{5\pi}{3})$	1A 1A 1A 1A 1A 1A+1A <hr style="width: 20px; margin-left: auto; margin-right: 0;"/>	-1 for each extraneous solution <hr style="width: 20px; margin-left: 0; margin-right: auto;"/>
5.	$kx^2 - 4x + 2k = 0$ $\alpha + \beta = \frac{4}{k}$ $\alpha\beta = 2$ $(a) \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \frac{16}{k^2} - 2(2) = \frac{16}{k^2} - 4$ $(b) \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{\frac{16}{k^2} - 4}{2} = \frac{8-4k^2}{k^2}$ $= \frac{8}{k^2} - 2$	1A 1A 1A 1M 1A 1M 1M <hr style="width: 20px; margin-left: 0; margin-right: auto;"/>	$\alpha + \beta = (-\frac{4}{k})^2 - 2(2)$ $= \frac{16}{k^2} - 4$ $\text{or } \frac{16-4k^2}{k^2}$ or equivalent.
6.	By symmetry, $\angle BAE = 30^\circ$ AS $OD \perp AB$, $\sin 30^\circ = \frac{1}{AO} \dots$ $\therefore AO = 2$ $AE = AO + OE$ $= 2 + 1 \dots$ $= 3$ $AB = AE$ $\therefore r = 3 \dots$	1A 1A 1A 1M 1A <hr style="width: 20px; margin-left: 0; margin-right: auto;"/>	



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P.3

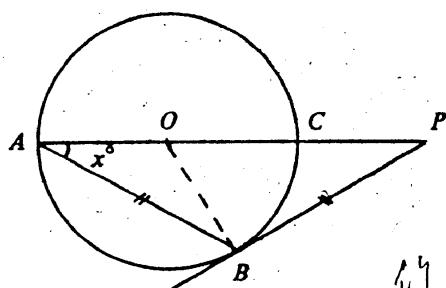
87 MATHS (SYLL A/B)

SOLUTION

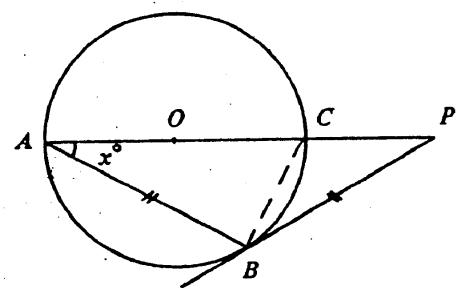
MARKS

REMARKS

7.



解法 1



Join OB.

As OA and OB are radii of the same circle,

$$\angle OBA = \angle PAB = x^\circ \dots \dots \dots$$

Since PB is a tangent,

$$\angle OBP = 90^\circ$$

Given that BA = BP

$$\angle BPA = \angle PAB = x^\circ \dots \dots \dots$$

$$x + x + x + 90 = 180$$

$$\begin{aligned} & \text{解法 1} \\ & 3x = 90 \\ & x = 30 \end{aligned} \quad \text{解法 2} \quad \begin{aligned} & \text{解法 3} \\ & \text{等腰直角三角形} \\ & \text{底角} = 45^\circ \end{aligned}$$

Alternatively:

1A Join BC.

As PB is a tangent,
 $\angle CBP = \angle PAB = x^\circ$.

1A Since AC is a diameter,
 $\angle ABC = 90^\circ$
etc.

1A

1A

1A

6

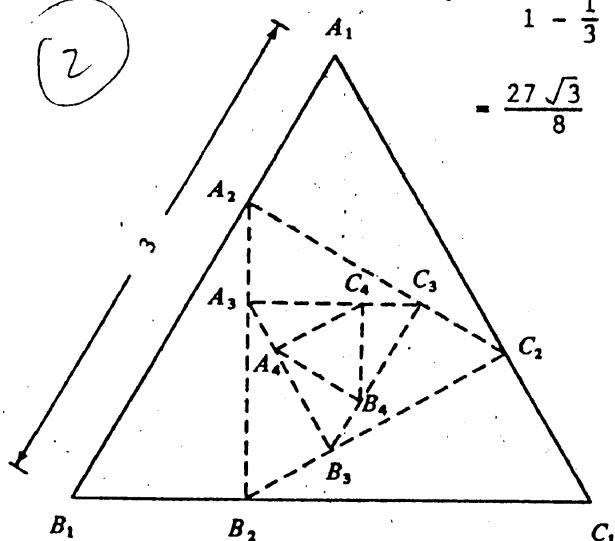
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P.5

87 MATHS (SYLL A/B)

SOLUTION	MARKS	REMARKS
<p>9. (a) (i) Capacity of hemispherical part</p> <p>(3) $\frac{1}{2} \times \frac{4}{3} \pi r^3$ 若將寫出有 $r^3 = \frac{3}{4}V$ 號 $= \frac{1}{6}(108\pi)$ 則只得 1M.</p> <p>$r^3 = 27$ $r = 3$</p> <p>Capacity of cylindrical part</p> <p>$= \pi r^2 h$ $= 9\pi h$</p> <p>$9\pi h = \frac{5}{6}(108\pi)$</p> <p>$h = 10$</p> <p>(ii) Volume of space = $\pi(3^2)(4)$</p> <p>(3) Volume of water = $108\pi - (\pi)(3^2)(4)$</p> <p>$= 72\pi \text{ cm}^3$</p>	<p>1M+1A</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>1A</p> <p>1M+1M</p> <p>1A</p>	<p>若寫 $r = \dots \text{ cm}$ 有它足</p> <p>1M for setting up eqn in r^3. 1A for correct eqn.</p> <p>若寫 $r = \dots \text{ cm}$ 有它足</p> <p>Alternatively: Volume $= \pi(3)^2(10-4) + \frac{108\pi}{6}$... 1M+1M $= 72\pi \text{ cm}^3$ 1A</p>
<p>(b)</p> <p>Let radius and depth of water be R and H.</p> <p>$\frac{1}{3}\pi R^2 H = 72\pi$ 無上管留 1M</p> <p>$R^2 H = 216$</p> <p>Capacity of vessel = $\frac{1}{3}\pi(2R)^2(2H)$</p> <p>$= \frac{8}{3}\pi R^2 H$</p> <p>$= \frac{8}{3}\pi \cdot (216)$</p> <p>$= 576\pi \text{ cm}^3$</p>	<p>1M</p> <p>1M</p> <p>1M</p> <p>1A</p>	<p>若無 $\frac{1}{3}\pi$ 得 1M</p> <p>-1 if unit not given</p>
<p>Alternatively:</p> <p>Since height of vessel = 2 X height of water Capacity of vessel = $2^3 \times 72\pi$ $= 576\pi \text{ cm}^3$</p>	<p>2M</p> <p>1A</p> <p>3</p>	<p>-1 if unit not given</p>

SOLUTION	MARKS	REMARKS
10. (a) Since the triangle is equilateral, $\angle A_1 = 60^\circ$, $T_1 = \frac{1}{2} (3)(3)(\sin 60^\circ)$ $= \frac{9\sqrt{3}}{4}$	1M 1A <hr/> 2	
(b) (i) Since $A_2B_1 = 2$, $B_1B_2 = 1$ and $\angle B_1 = 60^\circ$, $\angle B_1B_2A_2 = 90^\circ$ $\therefore A_2B_2 = \sqrt{3}$	1M 1A	Alternatively: By cosine rule, $(A_2B_2)^2 = 2^2 + 1^2 - 2(2)(1)\cos 60^\circ$ $= 3$ $\therefore A_2B_2 = \sqrt{3}$
(ii) $\triangle A_2B_2C_2$ and $\triangle A_1B_1C_1$ are similar. The ratio of their sides is $\sqrt{3} : 3$. $\therefore T_2 = \frac{9\sqrt{3}}{4} \left(\frac{\sqrt{3}}{3}\right)^2$ $= \frac{3\sqrt{3}}{4}$ 擔受 $\frac{\sqrt{3}}{4}$	1M 1A <hr/> 4	
(c) (i) The common ratio = $\frac{1}{3}$ (1M, \therefore 本題無需計算) (ii) $T_n = \frac{9\sqrt{3}}{4} \left(\frac{1}{3}\right)^{n-1}$ \nwarrow 此指 $n-1$	1M 1M	
(iii) $T_1 + T_2 + \dots + T_n = \frac{9\sqrt{3}}{4} \cdot \frac{1 - (\frac{1}{3})^n}{1 - \frac{1}{3}}$ $= \frac{27\sqrt{3}(1 - \frac{1}{3^n})}{8}$	1M 1A	
(iv) The sum to infinity = $\frac{\frac{9\sqrt{3}}{4}}{1 - \frac{1}{3}} = \frac{27\sqrt{3}}{8}$	1M 1A <hr/> 6	



SOLUTION

MARKS

REMARKS

11. (a) Consider $\triangle ADE$. By the cosine rule

$$\begin{aligned} AE^2 &= AD^2 + DE^2 - 2AD \cdot DE \cos \angle ADE \\ &= 3^2 + 2^2 - 12\cos 80^\circ (-= 10.91622) \end{aligned}$$

$\triangle ADE$ 有
角 80°

$$AE = 3.304 \text{ cm (correct to 3 d.p.)}$$

1M

correct use of formula

1A

1A

3

(b) Consider $\triangle ADE$ again. By the sine rule,

$$\begin{aligned} \frac{DE}{\sin \angle DAE} &= \frac{AE}{\sin \angle ADE} \quad \text{(使用公式时会用)} \\ \sin \angle DAE &= \frac{DE \sin \angle ADE}{AE} \\ (-= \frac{1.9696}{3.304} = 0.59613) \end{aligned}$$

$$\angle DAE = 36.593^\circ \text{ (correct to 3 d.p.)}$$

(1 答案有誤, 不給 1A)

2M

or cos rule

1A

Accept 36.593-36.594

3

$$\begin{aligned} (c) DG &= AD \sin \angle DAE \\ (-= 3 \sin 36.593^\circ) \quad &\text{不答 1A} \\ (-= (3)(0.59613)) \quad &\text{不答} \\ &= 1.788 \text{ cm (correct to 3 d.p.)} \end{aligned}$$

1M

or $\sin \angle DAE = \frac{DG}{AD}$

1A

2

$$\begin{aligned} (d) BD^2 &= AB^2 + AD^2 \\ BD &= \sqrt{18} \\ &= 4.243 \text{ cm (correct to 3 d.p.)} \end{aligned}$$

1M

1A

2

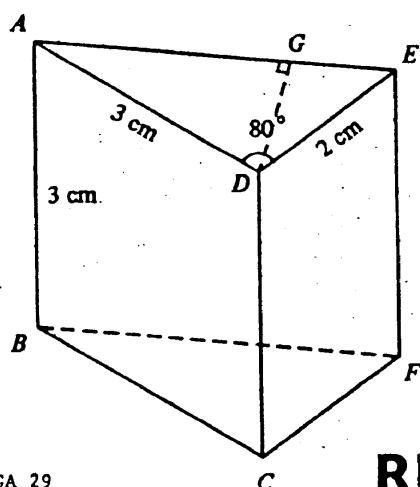
$$\begin{aligned} (e) \sin \angle DBG &= \frac{DG}{BD} \\ (-= \frac{1.788}{4.243} = 0.4214) \quad & \\ \therefore \angle DBG &= 24.923^\circ \text{ (correct to 3 d.p.)} \end{aligned}$$

1M

1A

Accept 24.920-24.940

2



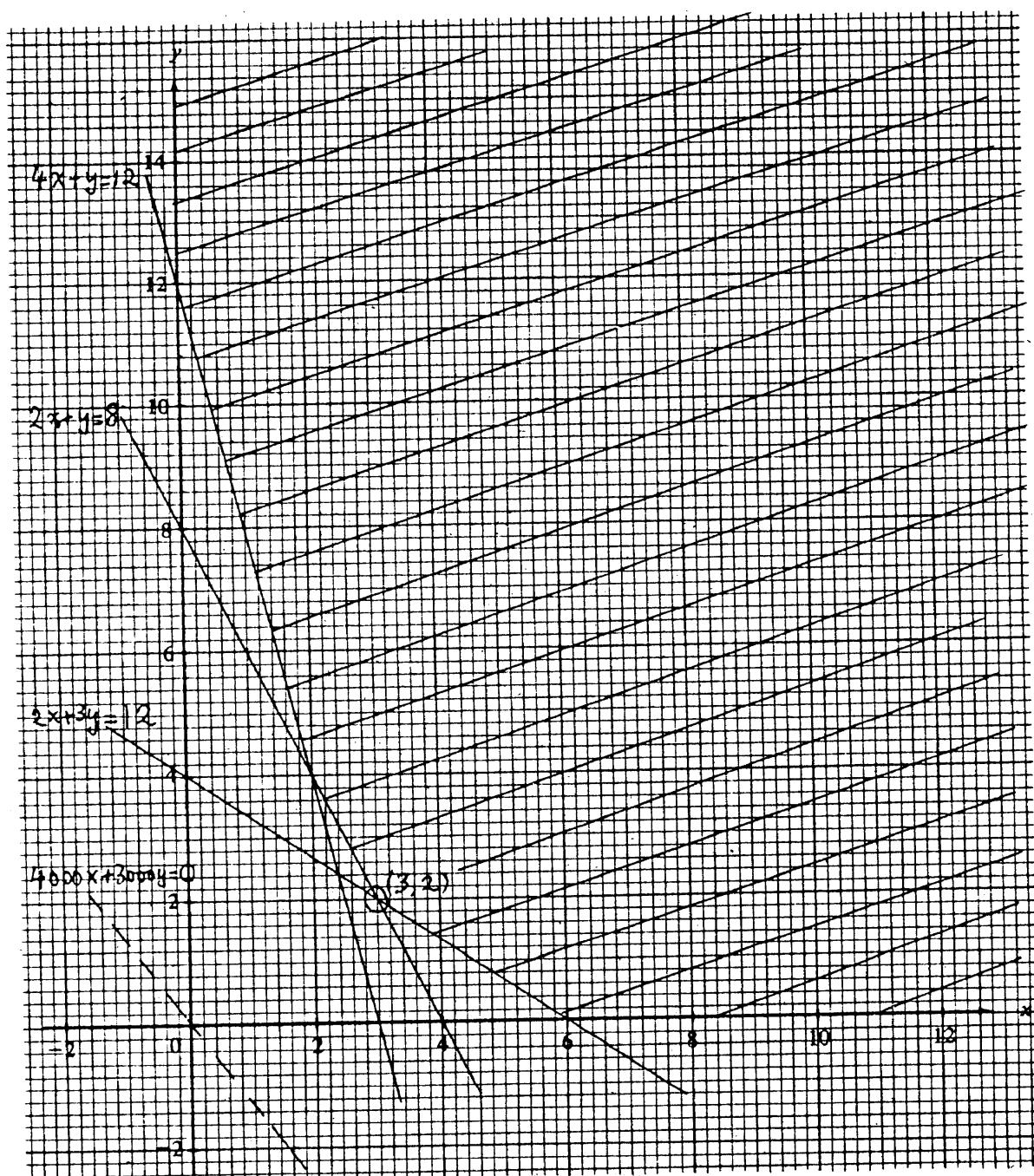
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87 MATHS (SYLL A/B)

P.8

SOLUTION	MARKS	REMARKS
12. (a) Given that $x \geq 0$ $y \geq 0$ $4000x + 6000y \geq 24000$ Considering Products B and C, $\begin{array}{l} 20000x + 5000y \geq 60000 \\ 6000x + 3000y \geq 24000 \end{array}$	1A 1A 2	Withhold 1A if '=' missing
(b) The constraints in (a) can be written as $\begin{array}{l} x \geq 0 \\ y \geq 0 \\ 2x + 3y \geq 12 \\ 4x + y \geq 12 \\ 2x + y \geq 8 \end{array}$		
The lines corresponding to the last 3 inequalities are shown on the graph paper. Shading the correct region. $4000x + 3000y = 0$	1A+1A +1A 3A 6	+1 unit at x,y axes -1 if shading not complete. -2 if only arrows used
(c) Cost of materials used = $4000x + 3000y$ (dollars) Drawing the line $4000x + 3000y = 0$ (or equivalent) $(斜率正确) 不需要太详细)$ The cost is least when $x = 3, y = 2$ and the least cost is 18 000 (dollars)	1M 1A 1A 1A 4	Candidates may also test all vertices of given region. Awarded only if region correct Point Cost (6,0) 24 000 (3,2) 18 000 (2,4) 20 000 (0,12) 36 000 4 最小收入 最小成本 Checking {, 1/3 c

12.



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P.10

87 MATHS (SYLL A/B)

SOLUTION	MARKS	REMARKS
<p>13. (a) The probability that the black ball is not drawn = $\frac{5}{6}$ (or $1 - \frac{1}{6} = \frac{5}{6}$)</p> <p>(2)</p>	2A	<p>Any value roundable to 0.83 P.P. if only answer is given. However, accept $P = 5/6$.</p> <p><u>2</u></p>
<p>(b) The probability that the black ball is drawn from P to Q in the 1st draw = $\frac{1}{6}$</p> <p>(4) After that, the probability that the black ball is not drawn from Q to R in the 2nd draw = $\frac{4}{5}$</p> <p>\therefore the probability that the black ball is in Q</p> $= \frac{1}{6} \times \frac{4}{5} = \frac{1}{6} + \frac{4}{5} = 1 + 1$ $= \frac{2}{15} \quad (= \frac{4}{30}) \quad \frac{1}{6} \times \frac{4}{5} \times = 1 + 1$	1A 1A <u>2A</u> <u>4</u>	
<p>(c) The probability that the black ball is drawn from Q to R = $\frac{1}{5}$</p> <p>(3) \therefore the probability that the black ball is in R</p> $= \frac{1}{6} \times \frac{1}{5} = \frac{1}{6} + \frac{1}{5} = 1 + 1$ $= \frac{1}{30} \quad (= 0.03) \quad \frac{1}{6} \times \frac{1}{5} = 1 + 1$	1A	<p><u>Alternatively:</u></p> <p>$1 - \frac{5}{6} = \frac{2}{15}$</p> <p>(1M) 2M</p>
<p>(d) The probability that a white ball is drawn from P to Q in the 1st draw = $\frac{3}{6} (= \frac{1}{2})$</p> <p>(3) After that, the probability that a white ball is drawn from Q to R in the 2nd draw = $\frac{1}{5}$</p> <p>\therefore the probability that all balls in R are white</p> $\text{white} = \frac{1}{2} \times \frac{1}{5} = 1 + 1 \quad 1 - (\frac{3}{6} \times \frac{1}{5} + \frac{1}{6} \times \frac{4}{5})$ $= \frac{1}{10} \quad \quad 1 - (\frac{3}{6} \times \frac{1}{5} + \frac{1}{6} \times \frac{4}{5})$ $= 0.1 \quad \frac{1}{10} = \frac{3}{30}$	1A 1A 1A <u>3</u>	<p>若不考虑所求概率 概率有误</p> <p>(PP-1)</p> <p>若所有球为白球 而有不在白色球 的全部被放</p>

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87 MATHS (SYLL A/B)

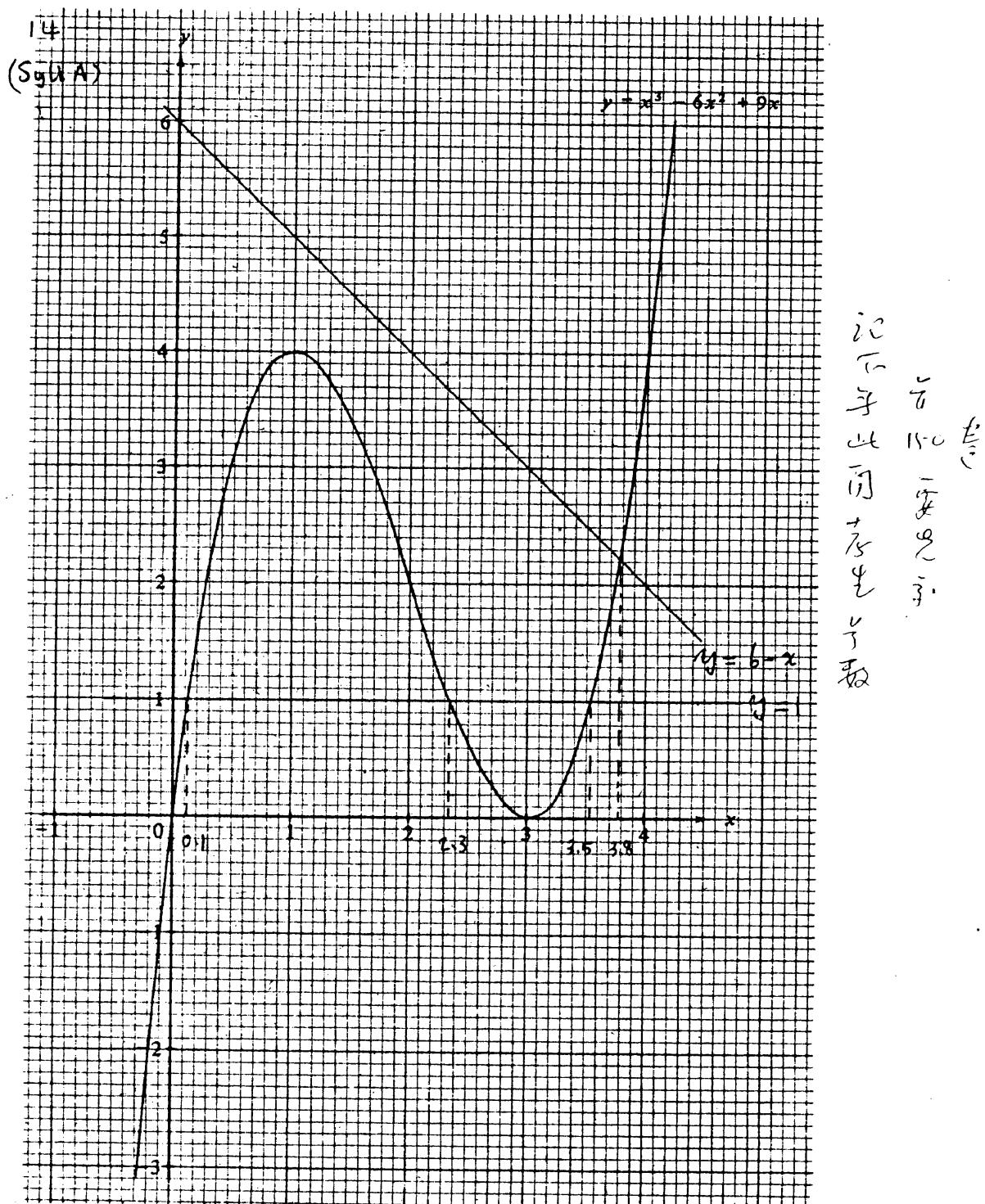
P.11

SYLLABUS	SOLUTION	MARKS	REMARKS								
(Syllabus A)											
14. (a) (i) $x^3 - 6x^2 + 9x - 1 = 0$	$\textcircled{3} \quad x^3 - 6x^2 + 9x = 1$	1M									
	Drawing the line $y = 1$, the roots of the given equation were found to be 0.1, 2.3 and 3.5 (correct to 1 d.p.).	1A+1A	1 mark for 2 correct answers								
	$\textcircled{3} \quad (ii) x^3 - 6x^2 + 10x - 6 = 0$	1M	for correct L.S.								
	$x^3 - 6x^2 + 9x = 6 - x \dots$	1A	for graph, 1 unit at (3,3), (4,2)								
	Drawing the line $y = 6 - x$, the root was found to be 3.8 (correct to 1 d.p.)	1A									
	$\textcircled{3} \quad \begin{array}{l} \text{graph} \\ \text{at } 3.7 \text{ is } -\frac{1}{2} \text{ of } 1A \end{array}$	6									
(b)	<table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <th>x</th> <th>$x^3 - 6x^2 + 10x - 6$</th> </tr> <tr> <td>3.76</td> <td>- (-0.068)</td> </tr> <tr> <td>3.77</td> <td>+ (-0.005)</td> </tr> <tr> <td>3.765</td> <td>- (-0.031)</td> </tr> </table> <p style="margin-left: 20px;">- change sign 1M</p>	x	$x^3 - 6x^2 + 10x - 6$	3.76	- (-0.068)	3.77	+ (-0.005)	3.765	- (-0.031)	1M	Change of sign, -ve for 3.765-3.769 May use graphical method
x	$x^3 - 6x^2 + 10x - 6$										
3.76	- (-0.068)										
3.77	+ (-0.005)										
3.765	- (-0.031)										
	$\therefore x = 3.77$ (correct to 2 d.p.)	1A									
		3									
(c)	Consider $x^3 - 6x^2 + 9x = k$ \leftarrow graph	1M									
	From the graph, if $0 < k < 4$,	1A+1A	-1 for ' $<$ ' if otherwise correct.								
	the line $y = k$ meets the curve $y = x^3 - 6x^2 + 9x$ at three distinct points.		may omit								
	$\therefore x^3 - 6x^2 + 9x - k = 0$ has three distinct roots.	3									

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P.12

87 MATHS (Syll A/B)



SOLUTION <u>(Syllabus B)</u>	MARKS	REMARKS
14. (a) Since $y \propto x$ and $z \propto \frac{1}{x}$, $y = k_1 x$ and $z = \frac{k_2}{x}$ (for some real k_1, k_2). $\therefore p = k_1 x + \frac{k_2}{x}$ Putting $x = 2, p = 7$, (or $x = 3, p = 8$) $7 = 2k_1 + \frac{k_2}{2}$ i.e. $4k_1 + k_2 = 14$ Putting $x = 3, p = 8$. $8 = 3k_1 + \frac{k_2}{3} \dots \dots \dots$ or $9k_1 + k_2 = 24$ Solving these two equations, $5k_1 = 10$ $k_1 = 2$ $k_2 = 6$ $\therefore p = 2x + \frac{6}{x}$ When $x = 4, p = 2(4) + \frac{6}{4}$ $= \frac{19}{2} \dots \dots \dots$	1A+1A 1M 1A 1A	Accept $y = kx, z = \frac{k}{x}$
(b) $2x + \frac{6}{x} < 13$	1M	
$2x^2 - 13x + 6 < 0$ (as $x > 0$)	1A	
$(2x - 1)(x - 6) < 0$		
$\therefore \frac{1}{2} < x < 6$	2A 4	-1 for ' \leq '